$Z_{AG} = \frac{V_{AG}}{I_A + k_0 \cdot 3I_O}$

$Z_{BG} = \frac{V_{BG}}{I_B + k_0 \cdot 3I_O}$

$R_f = 0, 1, 4 \Omega$

Diagram of electrical circuit with labels $Z_{line}$, $V_A, V_B, V_C$, $I_A, I_B, I_C$, and $Z_1, Z_2, Z_0$.
Quadrilateral Element

- More common on ground elements

Eline

65% - 75% of line

reactive reach setting

Rleft

Rright

2-8 times of reactive reach
\[ Z_{AG} = \frac{V_{AG}}{I_A + 3k_o I_o} \]

\[ \text{criteria} \]
\[ \text{Im}(Z_{AG}) < X_{set} \]
\[ -R_{set} < \text{Re}(Z_{AG}) < R_{set} \]
Alternative Implementation

\[ jX \cdot I \]

\[ \delta V \]

\[ V \]

\[ I_{pol} \]

\[ X_{reach} \]

\[ Z_{line} \cdot I \]

\[ r \cdot Z_{line} \cdot I - V \cdot I_{pol}^* \]

\[ \text{on the reactive reach limit} \]
For a fault,
\[ V = V_p \]
\[ I = I_p + k_0 I_z \]
If using negative-sequence current as polarizing current, \[ I_{pol} = I_z \]

Reactance Element

\[ m \left[ \frac{\text{Im}(V_{2.1}^* - V_{1.1}^* )}{\text{Im}(V_{1.1}^* - I_{1.1}^* )} \right] = 0 \]
\[ m \left| Z_{1.1} \right| = \text{Im}(V_{1.1}^* - I_{1.1}^* ) \]

\[ Z_{1.1} \text{ - positive sequence line impedance} \]
\[ \theta_{Z_{1.1}} \text{ - the angle of } Z_{1.1} \]
For A-G fault

\[ V = V_{AG}, \quad I = I_\phi + k_o \cdot 3I_0 \]

\[ I_\phi = I_A = \frac{3}{2} (I_{A2} + I_{A0}) \]

consider load current effects?

\[ I_A = I_1 + I_2 + I_0 \]

\[ I_1 = I_1\text{fault} + I_1\text{load} \]

\[ R_{cal} = \frac{\text{Im}[V \cdot (1 / Z_o \cdot I^*)]}{\text{Im}[I_\phi \cdot (1 / Z_o \cdot I^*)]} \]

**Resistance Element**
If the system is not homogeneous
- the angle of source ≠ angle of load impedance
- ratio of $Z_0$ to $Z_1$ differ

- If system is radial, no problem
- If remote source exists, there can be a problem

If considering tilt

$$m|Z_{1L}|$$

$$\frac{\text{Im}[V \cdot (I_{pol} \cdot 1/\hat{j}T^*)]}{\text{Im}[1/\hat{j}Z_{1L} \cdot I_{(pol)} \cdot 1/\hat{j}T^*]}$$

$\text{Tof}$ — zero sequence fault current

$I_{ol}$ — zero sequence left side fault current
ECE 526

Quadrilateral Distance Element Example

The impedances for the system below are given in secondary ohms. The zone 1 reach of the relay at BUS 1 is set to cover 85\% of the line.

![Diagram of electric system with impedances and relays](image)

\[ V_{secLL} := 120V \quad \text{Note that this is a line to line voltage.} \]

\[ Z_{L1} := 6\text{ohm} \cdot e^{j \cdot 85\text{deg}} \]
\[ Z_{S1} := 2.5\text{ohm} \cdot e^{j \cdot 85\text{deg}} \]
\[ Z_{R1} := j \cdot 3\text{ohm} \]
\[ \theta_{L1} := \arg(Z_{L1}) \quad \theta_{L1} = 85\text{-deg} \]

\[ Z_{L2} := Z_{L1} \]
\[ Z_{S2} := Z_{S1} \]
\[ Z_{R2} := Z_{R1} \]
\[ Z_{L0} := 3 \cdot Z_{L1} \]
\[ Z_{S0} := 3 \cdot Z_{S1} \]
\[ Z_{R0} := 3 \cdot Z_{R1} \]

A. With the breaker at Bus 2 open, calculate how much fault resistance can be present before the distance element is unable to see the a SLG fault on phase A in Zone 1 for faults at the following locations: (a) 10\% of the line, (b) 50\% of the line and (c) 80\% of the line if the relay is:

1. self-polarized
2. cross-polarized (use \(V_p\), \(V_c\))
3. positive sequence memory polarized (use the prefault source voltage)

AG fault
Define some useful quantities:

\[ Z_{1MAG} := |Z_{L1}| \quad Z_{1ANG} := \arg(Z_{L1}) \quad k := 0, 1 \ldots 719 \]

\[ MVA := 1000\text{kW} \quad \text{pu} := 1 \]

\[ a := 1 \cdot e^{j120\text{deg}} \quad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \]

\[ k_0 := \frac{Z_{L0} - Z_{L1}}{3 \cdot Z_{L1}} \quad k_0 = 0.667 \]

Sequence impedances to left of fault:

\[ Z_{\text{Left1}}(m) := Z_{S1} + m \cdot Z_{L1} \quad Z_{\text{Left1}}(0.5) = (0.479 + 5.479i) \Omega \]

\[ Z_{\text{Left2}}(m) := Z_{S1} + m \cdot Z_{L1} \quad Z_{\text{Left2}}(0.5) = (0.479 + 5.479i) \Omega \]

\[ Z_{\text{Left0}}(m) := Z_{S0} + m \cdot Z_{L0} \quad Z_{\text{Left0}}(0.5) = (1.438 + 16.437i) \Omega \]

\[ V_f := \frac{jV_{\text{secLL}}}{\sqrt{3}} \quad V_f = 69.282i \text{V} \]

Define a vector of \( R_f \) values (this is just for illustration, not the actual solution values)

\[ R_f := 0 \text{ohm}, 2.0 \text{ohm} \ldots 10.0 \text{ohm} \]
Zero sequence fault current as a function of fault resistance (recall that the $R_f$ vector starts at 0)

\[
I_0(m, R_f) := \frac{V_f}{Z_{Left1}(m) + Z_{Left2}(m) + Z_{Left0}(m) + 3 \cdot R_f}
\]

\[
I_1(m, R_f) := I_0(m, R_f)
\]

\[
I_2(m, R_f) := I_0(m, R_f)
\]

\[
V_1(m, R_f) := V_f - I_1(m, R_f) \cdot Z_{S1}
\]

\[
V_2(m, R_f) := 0 - I_2(m, R_f) \cdot Z_{S2}
\]

\[
V_0(m, R_f) := 0 - I_0(m, R_f) \cdot Z_{S0}
\]

\[
I_{ABC}(m, R_f) := A_{012} \cdot \begin{pmatrix}
I_0(m, R_f) \\
I_1(m, R_f) \\
I_2(m, R_f)
\end{pmatrix}
\]

\[
V_{ABC}(m, R_f) := A_{012} \cdot \begin{pmatrix}
V_0(m, R_f) \\
V_1(m, R_f) \\
V_2(m, R_f)
\end{pmatrix}
\]

B. Repeat part A with the breaker at Bus 2 closed. Comment on your results for both part A and part B.

- Update fault calculations:
  Sequence impedances to right of fault:

\[
Z_{R1} = 3i \Omega \\
Z_{R2} = 3i \Omega \\
Z_{R0} = 9i \Omega
\]

\[
Z_{Right1}(m) := Z_{R1} + (1 - m) \cdot Z_{L1}
\]

\[
Z_{Right1}(0.5) = (0.261 + 5.989i) \Omega
\]

\[
Z_{Right2}(m) := Z_{R1} + (1 - m) \cdot Z_{L1}
\]

\[
Z_{Right2}(0.5) = (0.261 + 5.989i) \Omega
\]
\[ Z_{\text{Right0}}(m) := Z_{R0} + (1 - m) \cdot Z_{L0} \quad \text{and} \quad Z_{\text{Right0}}(0.5) = (0.784 + 17.966i) \Omega \]

Thevenin equivalent (not counting fault resistance)

\[ Z_{\text{Thev1}}(m) := \left( \frac{1}{Z_{\text{Left1}}(m)} + \frac{1}{Z_{\text{Right1}}(m)} \right)^{-1} \quad \text{and} \quad Z_{\text{Thev1}}(0.5) = (0.19 + 2.863i) \Omega \]

\[ Z_{\text{Thev2}}(m) := \left( \frac{1}{Z_{\text{Left2}}(m)} + \frac{1}{Z_{\text{Right2}}(m)} \right)^{-1} \quad \text{and} \quad Z_{\text{Thev2}}(0.5) = (0.19 + 2.863i) \Omega \]

\[ Z_{\text{Thev0}}(m) := \left( \frac{1}{Z_{\text{Left0}}(m)} + \frac{1}{Z_{\text{Right0}}(m)} \right)^{-1} \quad \text{and} \quad Z_{\text{Thev0}}(0.5) = (0.571 + 8.588i) \Omega \]

Zero sequence fault current as a function of fault resistance (recall that the \( R_f \) vector starts at 0)

\[ I_{0F}(m, R_f) := \frac{V_f}{Z_{\text{Thev1}}(m) + Z_{\text{Thev2}}(m) + Z_{\text{Thev0}}(m) + 3 \cdot R_f} \]

\[ I_{1F}(m, R_f) := I_{0F}(m, R_f) \]

\[ I_{2F}(m, R_f) := I_{0F}(m, R_f) \]

Current divider to get the current through the relay

\[ I_{0B}(m, R_f) := I_{0F}(m, R_f) \cdot \frac{Z_{\text{Right0}}(m)}{Z_{\text{Left0}}(m) + Z_{\text{Right0}}(m)} \]

\[ I_{1B}(m, R_f) := I_{1F}(m, R_f) \cdot \frac{Z_{\text{Right1}}(m)}{Z_{\text{Left1}}(m) + Z_{\text{Right1}}(m)} \]

\[ I_{2B}(m, R_f) := I_{2F}(m, R_f) \cdot \frac{Z_{\text{Right2}}(m)}{Z_{\text{Left2}}(m) + Z_{\text{Right2}}(m)} \]
\[ V_{1B}(m, R_f) := V_f - I_{1B}(m, R_f) \cdot Z_{S1} \]
\[ V_{2B}(m, R_f) := 0 - I_{2B}(m, R_f) \cdot Z_{S2} \]
\[ V_{0B}(m, R_f) := 0 - I_{0B}(m, R_f) \cdot Z_{S0} \]
\[ I_{ABC_B}(m, R_f) := A_{012} \begin{pmatrix} I_{0B}(m, R_f) \\ I_{1B}(m, R_f) \\ I_{2B}(m, R_f) \end{pmatrix} \]
\[ V_{ABC_B}(m, R_f) := A_{012} \begin{pmatrix} V_{0B}(m, R_f) \\ V_{1B}(m, R_f) \\ V_{2B}(m, R_f) \end{pmatrix} \]

**Quadrilateral Element**

**Zone 1 Settings:**
\[ X_{A_{Z1}} := 0.65 \cdot |Z_{L1}| \quad X_{A_{Z1}} = 3.9 \, \Omega \]
\[ R_{A_{Z1}} := 8 \cdot X_{A_{Z1}} \quad R_{A_{Z1}} = 31.2 \, \Omega \]

Note that Zone 1 is set shorter than the fault at 80% This ratio assumes a short line.

- Plot the element characteristics for relay A
\[ Z_{L_{zone1}} := \begin{pmatrix} 0 \\ 0.65 \cdot Z_{L1} \end{pmatrix} \]
\[ X_{zone1} := \begin{pmatrix} 0.65 \cdot Z_{L1} \\ 0.65 \cdot Z_{L1} + R_{A_{Z1}} \end{pmatrix} \]
- Forward resistive reach

\[ R_{\text{zone1}} := \begin{pmatrix} R_{A \cdot Z1} \\ 0.65 \cdot Z_{L1} + R_{A \cdot Z1} \end{pmatrix} \]

- So far the characteristics above just you the X=0 as the lower boundary
- There is normally a directional element supervising the element, and this forms the lower boundary
- This is offset from the line impedance by 90 degrees:

\[ \theta_{\text{dir}} := \arg(Z_{L1}) - 90\text{deg} \quad \theta_{\text{dir}} = -5\cdot\text{deg} \quad \arg(Z_{L1}) = 85\cdot\text{deg} \]

- We need to define the boundaries of the triangle (this includes extending the resistive reach line downward)

\[ \text{Dir}_{Z1} := \begin{bmatrix} \left(-\frac{R_{A \cdot Z1}}{2} \cdot \cos(\theta_{\text{dir}}) \cdot e^{j \cdot \theta_{\text{dir}}}) \right) \\ R_{A \cdot Z1} \cdot \cos(\theta_{\text{dir}}) \cdot e^{j \cdot \theta_{\text{dir}}} \end{bmatrix} \]

\[ R_{\text{zone1}} := \begin{pmatrix} R_{A \cdot Z1} \cdot \cos(\theta_{\text{dir}}) \cdot e^{j \cdot \theta_{\text{dir}}} \\ 0.65 \cdot Z_{L1} + R_{A \cdot Z1} \end{pmatrix} \]

- We also need to extend the resistive reach to the left of the line impedance vector

\[ X_{\text{zone1}} := \begin{bmatrix} 0.65 \cdot Z_{L1} - \frac{R_{A \cdot Z1}}{2} \\ 0.65 \cdot Z_{L1} + R_{A \cdot Z1} \end{bmatrix} \]

\[ R_{Z1 \_L} := \begin{pmatrix} -\left(\frac{R_{A \cdot Z1}}{2} \cdot \cos(\theta_{\text{dir}}) \cdot e^{j \cdot \theta_{\text{dir}}}) \right) \\ 0.65 \cdot Z_{L1} - \frac{R_{A \cdot Z1}}{2} \end{pmatrix} \]
Plot of Zone 1 for quadrilateral element

- Calculate Tilt Angle (note that in this case I0 calculated for the relay is equal to the total I0 at the fault so there is no tilt since there is no remote infeed)

1. Relay A: 
   \[
   A_{at\ TA} := \frac{I_0(1.0, 0\ ohm)}{I_0(1.0, 0\ ohm)}
   \]
   \[
   |A_{at\ TA}| = 1
   \]
   \[
   T_A := \arg(A_{at\ TA}) \quad T_A = 0\text{-deg}
   \]
$X_{GA}(m,R_f) := \frac{\text{Im}\left[V_{ABC}(m,R_f)0(3\cdot I_0(m,R_f))\cdot e^{j\cdot T_A}\right]}{\text{Im}\left[\left(1e^{-j\cdot L_1}\right)\left[I_{ABC}(m,R_f)0 + k_0\cdot(3\cdot I_0(m,R_f))\right]\left(3\cdot I_0(m,R_f)\right)\cdot e^{j\cdot T_A}\right]}$

$R_{FA}(m,R_f) := \frac{\text{Im}\left[V_{ABC}(m,R_f)0\left(1e^{-j\cdot L_1}\right)\left[I_{ABC}(m,R_f)0 + k_0\cdot(3\cdot I_0(m,R_f))\right]\right]}{\text{Im}\left[\frac{3}{2}\left(I_2(m,R_f) + I_0(m,R_f)\right)\cdot \left(1e^{-j\cdot L_1}\right)\left[I_{ABC}(m,R_f)0 + k_0\cdot(3\cdot I_0(m,R_f))\right]\right]}$

$0.1 \cdot |Z_{L_1}| = 0.6 \Omega \quad 0.5 \cdot |Z_{L_1}| = 3 \Omega \quad 0.8 \cdot |Z_{L_1}| = 4.8 \Omega$

$X_{GA}(0.1,R_f):=R_{FA}(0.1,R_f) = X_{GA}(0.5,R_f):=R_{FA}(0.5,R_f) = X_{GA}(0.8,R_f):=R_{FA}(0.8,R_f) =$

<table>
<thead>
<tr>
<th>$X_{GA}$</th>
<th>$R_{FA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 Ω</td>
<td>0 Ω</td>
</tr>
<tr>
<td>0.6</td>
<td>2</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
</tr>
<tr>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td>0.6</td>
<td>8</td>
</tr>
<tr>
<td>0.6</td>
<td>10</td>
</tr>
</tbody>
</table>

- Add remote infeed
- Calculate Tilt Angle (note that in this case I0 calculated for the relay is equal to the total I0 at the fault
  1. Relay A:

  $A_{at \_ T_B} := \frac{I_{0F}(1.0,0\text{ohm})}{I_{0B}(1.0,0\text{ohm})}$

  $|A_{at \_ T_B}| = 3.831 \quad T_B := \arg(A_{at \_ T_B}) \quad T_B = -3.696$ deg
\[
\begin{align*}
X_{GB}(m, R_F) &= \frac{\text{Im}(V_{ABC}(m, R_F))}{\text{Im}\left[\left(1 + j\theta_{L1}\right)\frac{V_{ABC}(m, R_F)}{j e^{j\frac{\pi}{2}} + \left(3\log(m, R_L) + k_0\left(3\log(m, R_L)\right)^2\right)}\right]} \\
R_{FB}(m, R_F) &= \frac{\text{Im}\left[1 - \frac{V_{ABC}(m, R_F)}{j e^{j\frac{\pi}{2}} + \left(3\log(m, R_L) + k_0\left(3\log(m, R_L)\right)^2\right)}\right]}{\text{Im}\left[V_{ABC}(m, R_F)\right]}
\end{align*}
\]

<table>
<thead>
<tr>
<th>(m)</th>
<th>(R_F)</th>
<th>(X_{GB}(0.8, R_F))</th>
<th>(R_{FB}(0.8, R_F))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8 (\Omega)</td>
<td>5.491</td>
<td>10.962</td>
<td>16.473</td>
</tr>
<tr>
<td>4.82 (\Omega)</td>
<td>4.964</td>
<td>5.046</td>
<td>5.12</td>
</tr>
<tr>
<td>5.21 (\Omega)</td>
<td>5.21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
Z_{GB}(m, R_F) = R_{FB}(m, R_F) + j X_{GB}(m, R_F)
\]

- Now include the effect of the tilt for faults at different points on the line. Note that the characteristic tilt up for faults closer than the location where the tilt was calculated. A downward slope is to be avoided.
$\text{Real}(Z_{\text{L zone1}}), \text{Real}(Z_{\text{zone1}}), \text{Real}(Z_{\text{R zone1}}), \text{Real}(Z_{\text{Rz1 L}}), \text{Real}(Z_{\text{GB}(1,R_f)}), \text{Real}(Z_{\text{GB}(0.8,R_f)}), \text{Real}(Z_{\text{GB}(0.5,R_f)}), \text{Real}(Z_{\text{GB}(0.1,R_f)})$
Quadrilateral Element Calculations for Non-Homogeneous Systems

- Voltage at the fault point for SLG fault:

\[ V_{AF} = V_{1f} + V_{2f} + V_{0f} = I_{AF} \cdot R_F = 3 \cdot I_{0F} \cdot R_F \]

- The voltage at the relay is shifted by the voltage drop across the transmission line between the fault point and the relay.

\[ V_{A_{\text{relay}}} = V_{1R} + V_{2R} + V_{0R} + V_{AF} = V_{1R} + V_{2R} + V_{0R} + 3 \cdot I_{0F} \cdot R_F \]

Rewriting this:

\[ V_{A_{\text{relay}}} = m \cdot Z_{L1} \cdot I_{1R} + m \cdot Z_{L1} \cdot I_{2R} + m \cdot Z_{L0} \cdot I_{0R} + 3 \cdot I_{0F} \cdot R_F \]

or

\[ V_{A_{\text{relay}}} = m \cdot Z_{L1} \left( I_{1R} + I_{2R} + \frac{Z_{L0}}{Z_{L1}} \cdot I_{0R} \right) + 3 \cdot I_{0F} \cdot R_F \]

\[ V_{A_{\text{relay}}} = m \cdot Z_{L1} \left( I_{1R} + I_{2R} + I_{0R} + \frac{Z_{L0}}{Z_{L1}} \cdot I_{0R} - I_{0R} \right) + 3 \cdot I_{0F} \cdot R_F \]

Now take advantage of known definitions:

\[ V_{A_{\text{relay}}} = m \cdot Z_{L1} \cdot (I_{AR} + 3 \cdot I_{0R} \cdot k_0) + 3 \cdot I_{0F} \cdot R_F \]

where \( k_0 = \frac{Z_{L0} - Z_{L1}}{3 \cdot Z_{L1}} \)

or in terms of ground currents

\[ V_{A_{\text{relay}}} = m \cdot Z_{L1} \cdot (I_{AR} + I_{G_{\text{relay}}} \cdot k_0) + I_{G_{\text{total}}} \cdot R_F \]
• Challenges
  1. We don't know \( m \)
  2. We don't know \( I_{G,\text{total}} \) at the local relay
  3. Have 2 equations and 3 unknowns

\[
V_{AR,\text{relay}} = m \cdot |Z_{L1}| \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0) + I_{G,\text{null}} \cdot R_F
\]

• Multiply this equation by complex conjugate of the relay current rotated by the line angle

\[
V_{AR} \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0) = m \cdot |Z_{L1}| \cdot \left( |I_{AR} + I_{G,\text{relay}} \cdot k_0|^2 \right) + I_{G,\text{total}} \cdot R_F \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0)
\]

• Equate the imaginary parts:

\[
\text{Im} \left[ V_{AR} \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0) \right] = \text{Im} \left[ I_{G,\text{total}} \cdot R_F \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0) \right]
\]

Note that: \( I_{G,\text{total}} = I_{G,\text{relay}} + I_{G,\text{remote}} = (1 + \varepsilon) \cdot I_{G,\text{relay}} \)

• This is a phasor sum
• In a homogeneous system, \( \varepsilon \) is real
• Otherwise it is complex

\[
\text{Im} \left[ (1 + \varepsilon) \cdot I_{G,\text{relay}} \cdot R_F \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0) \right] = \text{Im} \left[ V_{AR} \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0) \right]
\]

\[
R_F \cdot (1 + \varepsilon) = \frac{\text{Im} \left[ V_{AR} \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0) \right]}{\text{Im} \left[ (I_{G,\text{relay}}) \cdot e^{j \cdot \theta_{L1}} \cdot (I_{AR} + I_{G,\text{relay}} \cdot k_0) \right]}
\]
- ε varies with respect $Z_s$, $Z_r$ and m
- As a result the relay at one end can't measure accurate $R_F$
- The important part is that it causes the reactive reach equation to tilt up or down
- Calculate tilt and add as a correction factor.